8] Derive Gibbs phase rule thermodynamically

Let us Consider a heterogeneous System Containing C components ($C_1, C_2 \ldots C_i$) distributed among P phases ($P_1, P_2 \ldots P_i$)

Degree of freedom = Total no of Variables — Number of Variables defined by the System. → (1)

i) Calculation of total no of Variables;

→ If a phase Contains three Components two Concentration terms should be known to define a System Completely. In general, if a ~~cont~~ phase Contains C components, (C−1) Variables are necessary to define the system.

→ For P phases, total no of Variables necessary to define the System Completely will be P(C−1).

Total no of Variables = $P(C-1)+2$ → (2)

ii) Number of Variables

when a System is in eqm, Chemical potential of a Component is the Same in all the P Phases.

$(\mu_i)_A = (\mu_i)_B = (\mu_i)_C$.

These are the two eqns

i) $(\mu_i)_A = (\mu_i)_B$ ii) $(\mu_i)_B = (\mu_i)_C$

In General, for a System of P phases, $(P-1)$ equations are known for each Component.

Hence, for a System of P Phases and C components no of equations will be $C(P-1)$.

$\therefore$ No of Variables $= C(P-1)$ ---> ③

Substitute eqn ② & ③ in eqn ①

$\therefore$ Degree of Freedom (F) $= [P(C-1)+2] - [C(P-1)]$

$$= CP - P + 2 - [CP - C]$$

$$= \cancel{CP} - P + 2 - \cancel{CP} + C$$

$$\boxed{F = C - P + 2}$$